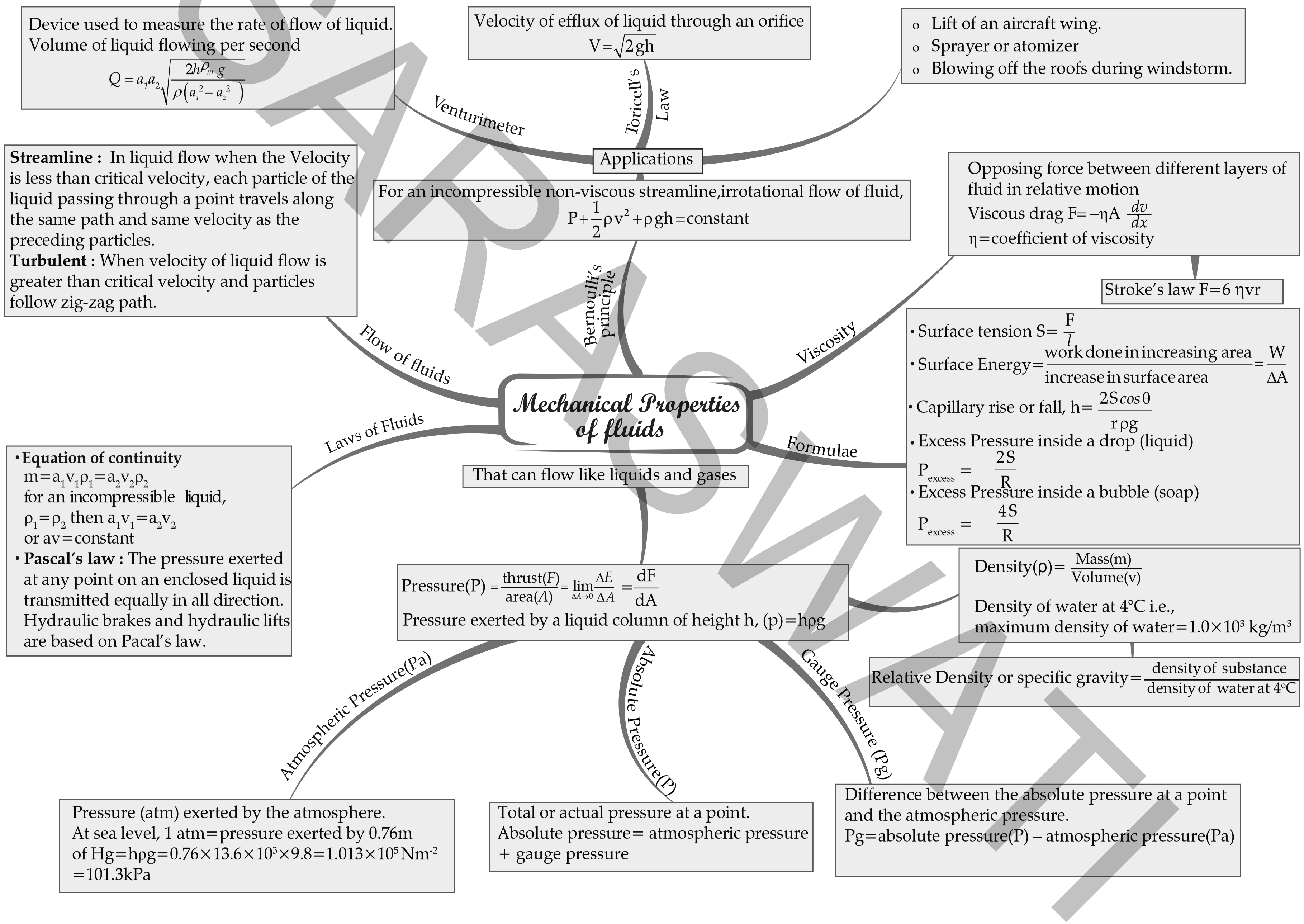
3 Mechanical Properties of Fluids

Mechanical Properties of Fluids

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure.
Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity.Bernoulli's theorem and its applications.
Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise.

MIND MAP : LEARNING MADE SIMPLE



FLUID MECHANICS

1. FLUID MECHANICS

- The liquids and gases together are termed as fluids, in other words, we can say that the substances which can flow are termed as fluids.
- We assume fluid to be incompressible (i.e., the density of liquid is independent of variation in pressure and remains constant) and non-viscous (i.e. the two liquid surfaces in contact are not exerting any tangential force on each other).

1.1 Fluid Statics

1.1.1 Fluid Pressure

Pressure p at every point is defined as the normal force per unit area.

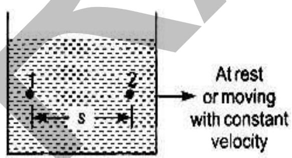
$$p = \frac{dF_{\perp}}{dA}$$

The SI unit of pressure is the Pascal and 1 Pascal = 1 N/m^2

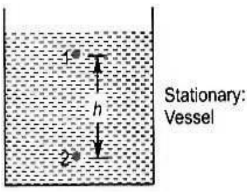
- Fluid force acts perpendicular to any surface in the fluid, no matter how that surface is oriented. Hence pressure, has no intrinsic direction of its own, it is a scalar.

Pressure

(a) Pressure at two points in a horizontal plane or at same level when the fluid is at rest or moving with constant velocity is same.

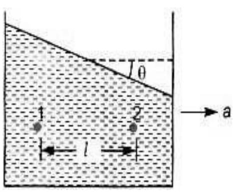


(b) Pressure at two points which are at a depth separation of h when fluid is at rest of moving with constant velocity is related by the expression



$p_2 - p_1 = \rho gh$, where ρ is the density of liquid.

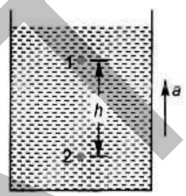
(c) Pressure at two points in a horizontal plane when fluid container is having some constant horizontal acceleration are related by the expression



$p_1 - p_2 = l\rho a$

and tan θ = a/g, where θ is the angle which the liquid's free surface is making with horizontal.

(d) Pressure at two points within a liquid at vertical separation of *h* when the liquid container is accelerating up are related by expression



$p_2 - p_1 = \rho (g + a) h$

If container is accelerating down, then $p_2 - p_1 = \rho(g - a) h$.

1.1.2 Atmospheric Pressure

- It is the pressure of the earth's atmosphere. Normal atmospheric pressure at sea level (an average value) is 1 atmosphere (atm) that is equal to 1.013×10^5 Pa.
- The excess pressure above atmospheric pressure is called gauge pressure, and total pressure is called absolute pressure.
- Barometer is a device used to measure atmospheric pressure while U–tube manometer or simply manometer is a device used to measure the gauge pressure.

1.1.3 Pascal's Law

- A change in the pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.
- There are a lot of practical applications of Pascal's law one such application is hydraulic lift.

1.1.4 Archimedes Principle

- When a body is partially or fully dipped into a fluid, the fluid exerts contact force on the body. The resulatant of all these contact forces is called buoyant force (upthrust).
- F = weight of fluid displaced by the body.
- This force is called buoyant force and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of the displaced fluid.

F = Vσg

where, v = volume of liquid displaced

σ = density of liquid.

- Apparent decrease in weight of body = upthrust = weight of liquid displaced by the body.
- Floation :

(a) A body floats in a liquid if the average density of the body is less than that of the liquid.

(b) The weight of the liquid displaced by the immersed part of body must be equal to the weight of the body.

(c) The centre of gravity of the body and centre of buoyancy must be along the same vertical line.

1.2 Fluid Dynamics

- **Steady Flow (Stream Line Flow)**

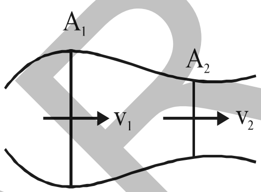
  The flow in which the velocity of fluid particles crossing a particular point is the same at all the times. Thus, each particle takes the same path as taken by a previous particle through that point.

- **Line of flow**

  It is the path taken by a particle in flowing liquid. In case of a steady flow, it is called streamline. Two steamlines can never intersect.

1.2.1 Equation of Continuity

In a time Δt, the volume of liquid entering the tube of flow in a steady flow is $A_1 V_1 \Delta t$. The same volume must flow out as the liquid is incompressible. The volume flowing out in Δt is $A_2 V_2 \Delta t$.



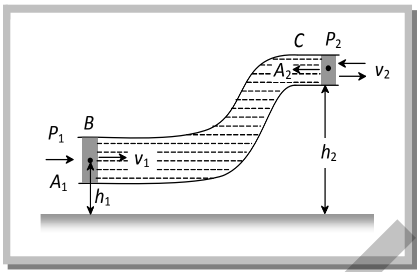
- $A_1 V_1 = A_2 V_2$
- mass flows rate = ρAV

  (where ρ is the density of the liquid.)

1.2.2 Bernoulli's Theorem

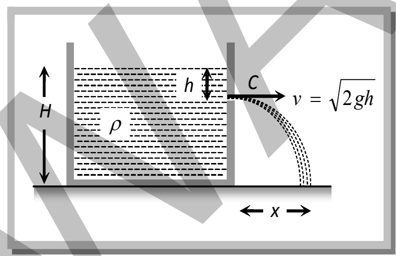
In a stream line flow of an ideal fluid, the sum of pressure energy per unit volume, potential energy per unit volume and kinetic energy per unit volume is always constant at all cross section of the liquid.

$$P+\rho gh+\frac{\rho V^2}{2}=\text{Constant}$$



- Bernoulli's equation is valid only for incompressible steady flow of a fluid with no viscosity.
- Application of Bernoulli's Theorem.

(a) Velocity of Efflux



Let us find the velocity with which liquid comes out of a hole at a depth h below the liquid surface.

Using Bernoulli's theorem,

$$P_A+\frac{1}{2}\rho V_A^2+\rho gh_A=P_B+\frac{1}{2}\rho V_B^2+\rho gh_B$$

$$\Rightarrow \quad P_{atm}+\frac{1}{2}\rho V_A^2+\rho gh=P_{atm}+\frac{1}{2}\rho V^2+0$$

(Note: $P_B = P_{atm}$, because we have opened the liquid to atmosphere)

$\Rightarrow$ $V^2 = V_A{}^2 + 2gh$

Using equation of continuity

$AV_A = aV$

A: area of cross-section of vessel

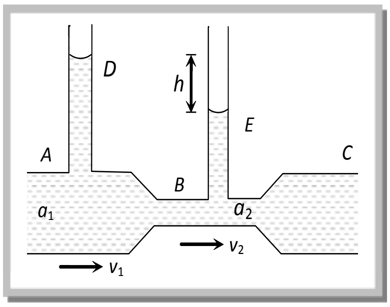
a: area of hole

$$\Rightarrow \quad V^2=\frac{a^2}{A^2}V^2+2gh$$

$$\Rightarrow \quad V=\frac{\sqrt{2gh}}{\sqrt{1-a^2/A^2}}\approx\sqrt{2gh} \qquad \text{(if the hole is very small)}$$

(b) Venturi Meter

This is an instrument for measuring the rate of flow of fluids.



If P_A is pressure at A and P_B is pressure at B,

$P_A - P_B = h\rho g$ [h : difference of heights of liquids of density ρ in vertical tubes]

If V_1 is velocity at A and V_2 is velocity at B

$Q = A_1 V_1 = A_2{}_{\ 2}$ [equation of continuity]

$$P_A + \rho \frac{V_1^2}{2} = P_B + \rho \frac{V_2^2}{2} \quad \text{[Bernoulli's Theorem]}$$

$$\Rightarrow \quad V_2^2 - V_1^2 = \frac{2}{\rho}(P_A - P_B) = \frac{2}{\rho} h\rho g$$

$$\Rightarrow \quad \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2hg \quad (Q = AV)$$

$$\Rightarrow \quad Q = A_1 A_2 \sqrt{\frac{2hg}{A_1^2 - A_2^2}}$$

1.3 Viscosity

The property of a fluid by virtue of which it opposes the relative motion between its different layers is known as viscosity and the force that is into play is called the viscous force.

Viscous force is given by :

$$F = -\eta A \frac{dv}{dx}$$

where η is a constant depending upon the nature of the liquid and is called the coefficient of viscosity and velocity gradient = dv/dx

S.I. unit of coefficient of viscosity is Pa.s or Nsm^{-2}.

CGS unit of viscocity is poise. (1 Pa.s = 10 Poise)

1.3.1 Stoke's Law

- When a solid moves through a viscous medium, its motion is opposed by a viscous force depending on the velocity and shape and size of the body.
- The viscous drag on a spherical body of radius r, moving with velocity v, in a viscous medium of viscosity η is given by

  $F_{viscous} = 6\pi\eta rv$

  This relation is called Stoke's law.
- **Importance of Stoke's law**

(a) This law is used in the determination of electronic charge with the help of milikan's experiment.

(b) This law accounts the formation of clouds.

(c) This law accounts why the speed of rain drops is less then that of a body falling freely with a constant velocity from the height of clouds.

(d) This law helps a man coming down with the help of a parachute.

1.3.2 Terminal Velocity

It is maximum constant velocity acquired by the body while falling freely in a viscous medium.

$$v_r = \frac{2r^2(\rho - \rho_0)g}{9\eta}$$

1.3.3 Poiseuille's Formula

Poiseuille studied the stream-line flow of liquid in capillary tubes.

Volume of liquid coming out of tube per second in given by

$$= \frac{\pi P r^4}{8\eta\ell}$$

1.3.4 Reynold Number

- The stability of laminar flow is maintained by viscous forces. It is observed, however that laminar or steady flow is disrupted it the rate of flow is large. Irregular, unsteady motion, turbulence, sets in at high flow rates.
- Reyonlds defined a dimensionless number whose value gives one an approximate idea, whether the flow rate would be turbulent.

  This number, called the Reynolds number R_e is defined as,

$$R_e = \frac{\rho v D}{\eta}$$

where, ρ = the density of the fluid flowing with a speed v.

D = the diameter of the tube.

η = the coefficient of viscosity of the fluid.

- It is found that flow is streamline or laminar for R_e less than 1000. The flow is turbulent for $R_e > 2000$. The flow becomes unsteady for R_e between 1000 and 2000.

1.4 Surface Tension

The surface tension of a liquid is defined as the force per unit length in the plane of the liquid surface at right angles to either side of an imaginary line drawn on that surface.

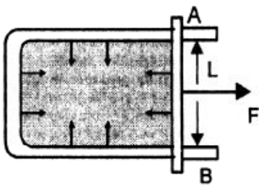
So, $S = \frac{F}{\ell}$ where S = surface tension of liquid.

Unit of surface tension in MKS system : N/m, J/m^2

CGS system : Dyne/cm, erg/cm^2

1.4.1 Surface Energy

In order to increase the surface area, the work has to be done over the surface of the liquid. This work done is stored in the liquid surface as its potential energy. Hence the surface energy of a liquid can be defined as the excess potential energy per unit area of the liquid surface.



$W = S\Delta A$, where ΔA = increase in surface area.

1.4.2 Excess Pressure

- Excess pressure in a liquid drop or bubble in a liquid is

$$P = \frac{2T}{R}$$

- Excess pressure in a soap bubble is $P = \frac{4T}{R}$

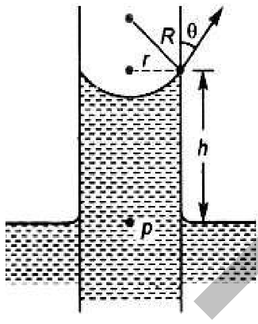
(because it has two free surfaces)

1.4.3 Angle of Contact

- The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is called the angle of contact.
- If the glass plate is immersed in mercury, the surface is curved and the mercury is depressed below. Angle of contact is obtuse for mercury.
- If the plate is dipped in water with its side vertical, the water is drawn-up along the plane and assumes the curved shape as shown. Angle of contact is acute for water.

1.4.4 Capillary Tube and Capillarity Action

- A very narrow glass tube with fine bore and open at both ends is known as capillary tube. When a capillary tube in dipped in a liquid, then liquid will rise or fall in the tube, this action is termed as capillarity.



$$h = \frac{2S\cos\theta}{r\rho g} = \frac{2S}{R\rho g}$$

where, S = surface tension,

θ = angle of contact,

r = radius of capillary tube,

R = radius of meniscus, and

ρ = density of liquid.

- **Capillary rise in a tube of insufficient length :**

If the actual height to which a liquid will rise in a capillary tube is 'h' then a capillary tube of length less than 'h' can be called a tube of "insufficient length".

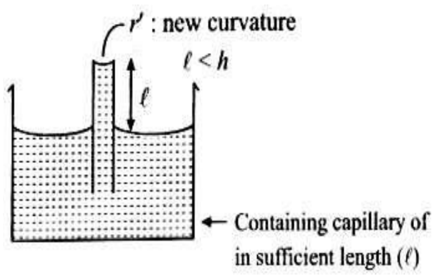
In such a case, liquid rises to the top of the capillary tube of length l ($l < h$) and adjusts the radius of curvature of its meniscus until the excess pressure is equalised by the pressure of liquid column of length l. (Note liquid does not overflow).

$$\Rightarrow \quad \frac{2\sigma}{r'} = \ell\rho g \quad \text{... (i)}$$

If r were the actual radius of curvature,

$$\Rightarrow \quad \frac{2\sigma}{r} = h\rho g \quad \text{...(ii)}$$

Comparing (i) and (ii)



$$\frac{2\sigma}{\rho g} = \ell r' = hr$$

$\Rightarrow \quad r' = \frac{hr}{\ell}$ i.e. radius of curvature r′ can be calculated.

	Adhesion > Cohesion	Adhesion = Cohesion	Adhesion < Cohesion
1.	Liquid will wet the solid.	Critical.	Liquid will not wet the solid.
2.	Meniscus is concave.	Meniscus is plane.	Meniscus is convex.
3.	Angle of contact is acute ($\theta = 90°$).	Angle of contact is 90°.	Angle of contact is obtuse ($\theta = 90°$).
4.	Pressure below the meniscus is lesser than above it by (2T/r), i.e. $P = P_0 - \frac{2T}{r}$.	Pressure below the meniscus is same as above it, i.e. $P = P_{0.}$	Pressure below the meniscus more then above it by (2T/r), i.e., $P = P_0 + \frac{2T}{r}$.
5.	In capillary there will be ascend.	No capillarity.	In capillary there will be descend.

QUESTION ALIKE

Very Short Answer Type Questions :

1. A block of wood is floting in a lake. What is apparent weight of the block?

Sol. The apperent weight of the floating block is equal to zero because weight of the block is balanced by force of buoyancy.

2. A boat carrying a large number of stones is floating in a water tank. What will happen to water level if the stones are unloaded into water?

Sol. Pascal's Law

The water level falls because volume of the water displaced by stones in water will be less than the volume of water displaced when stones are in the boat.

3. Why is pressure a scalar quantity?

Sol. The pressure acting on an area takes only the normal component of force acting on the area, and not the force vector.

4. What is the principle behind equation of continuity?

Sol. Conservation of mass.

5. What is the dimensional formula of Reynolds number?

Sol. $[M^0L^0T^0]$

6. What do you mean by critical Reynolds number?

Sol. The exact value of Reynolds number above which the flow of a liquid changes from streamline to turbulent is called critical Reynolds number.

7. What will happen when a body of density $2d$ is released at the surface of a liquid of density $\frac{d}{2}$?

Sol. The body sinks down in the liquid.

8. What is the shape of a liquid surface, in a container when the angle of contacts are respectively acute and obtuse?

Sol. Concave and convex.

9. How does the terminal velocity of a liquid drop falling down change when its radius increases by a factor 1.5?

Sol. $v_T \propto r^2$

$$\Rightarrow \quad \frac{v_T}{v_T'} = \left(\frac{r}{1.5r}\right)^2$$

$$\Rightarrow \quad v_T' = 2.25\ v_T$$

Thus, terminal velocity increases by a factor 2.25.

10. What does it imply if critical Reynolds number for two different fluids is almost same?

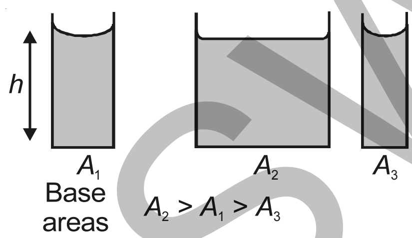
Sol. They flow through the pipes of same diameter.

11. In which type of capillary (material of capillary) will water descend and not rise?

Sol. A capillary made of paraffin wax.

Short Answer Type Questions :

12. A liquid is filled into three vessels upto same height as shown



For which vessel is the force exerted by the liquid at the bottom is maximum.

Sol. Since liquid is filled upto the same height

$\therefore\ P_1 = P_2 = P_3$

Respective forces exerted at the bottoms are

$F_1 = P_1A_1$ $\qquad$ $F_2 = P_2A_2$ $\qquad$ $F_3 = P_3A_3$

Since $A_2 > A_1 > A_3$

$\Rightarrow \quad F_2 > F_1 > F_3$

Thus liquid exerts maximum force at the bottom of the flask 2.

13. A 600 g of solid cube having an edge length of 10 cm floats in water. How much volume of the cube is inside water? (Given density of water = 10^3 kg m^{-3})

Sol. The upward buoyant force balances the weight of the cube. Let the volume of cube inside the water be V, then

$mg = V\rho g$

(600 g)(10 ms^{-2}) = V(density of water)(10 ms^{-2})

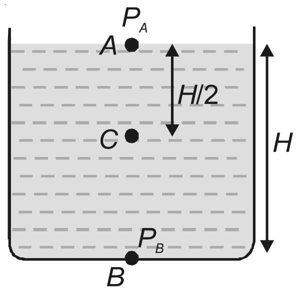
$\Rightarrow$ 0.6 kg = $V \times$ (10^3 kg m^{-3})

$$\Rightarrow V = \frac{0.6}{10^3}$$

$= 6 \times 10^{-4}$ m^3

$= 600$ cm^3

14. In the shown figure, the pressure at points A and B are respectively P_A and P_B. Prove that pressure at point C is $\left(\frac{P_A + P_B}{2}\right)$.



Sol. At A, pressure = P_A

At B, $P_B = P_A + \rho g H$

At C, $P_C = P_A + \left(\rho g \frac{H}{2}\right)$

Now $P_A + P_B = P_A + P_A + \rho g H$

$$= 2\left(P_A + \rho g \frac{H}{2}\right)$$

$= P_A + P_B = 2(P_C)$

$\Rightarrow \quad P_C = \frac{P_A + P_B}{2}$

15. Water does not come out of the dropper unless its rubber bulb is pressed hard. Why?

Sol. Water is held inside the dropper against the atmospheric pressure. When the rubber bulb is pressed, pressure on water increases and pushes it out against the atmospheric pressure.

16. What happens if water is used in barometers instead of mercury?

Sol. (i) Density of water is 10^3 kg m^{-3} while that of mercury is 13.6×10^3 kg m^{-3}. Thus a barometer with water will require a tube of about 11 m to measure the atmospheric pressure. This is impractical.

(ii) Also water wets the barometer tube while mercury does not.

17. A barometer kept in an elevator accelerating downwards reads 76 cm of Hg. What is the air pressure inside the elevator?

Sol. Since the elevator is accelerating downwards

Net acceleration as experienced by the barometer tube = $g - a$

The height of mercury corresponding to pressure inside the elevator,

$$h = \frac{76 \times 13.6\,(g - a)}{13.6 \times g}$$

18. Why don't two streamlines intersect?

Sol. The tangent at any point of a streamline gives the direction of flow of fluid particle at that point. If two streamlines intersect at a point, two tangents can be drawn at that point. Which means the oncoming fluid particle can go one way or the other. The flow will then no more be streamlined.

19. Why does the velocity decrease when water flow in a narrow pipe enters a broader pipe?

Sol. According to the equation of continuity,

Av = constant

where A = cross-sectional area of pipe

v = fluid velocity

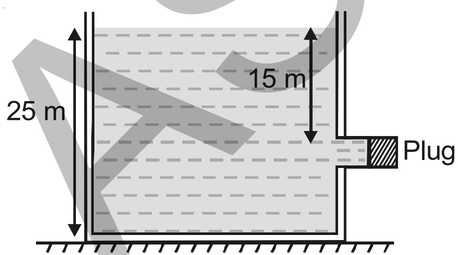
$$\Rightarrow v \propto \frac{\text{constant}}{A}$$

Therefore, as the cross-sectional area available to water increases, its velocity decreases.

20. What is the effect of temperature on viscosity and surface tension of a liquid?

Sol. Viscosity and surface tension both decrease with increase in temperature of a liquid. For gases viscosity increases with increase in temperature.

21. The water in a reservoir is 25 m deep. A horizontal pipe 5 cm in diameter passes through the reservoir 15 m below the water surface as shown. A plug secures the pipe opening. Find the force of friction between the plug and the pipe wall. [Take $g = 10 \text{ m s}^{-2}$]



Sol. Since the plug is stationary, static friction acts between the plug and the pipe wall.

$\Rightarrow$ Static friction, f_s = Force of push by the water

= Gauge pressure × Area of cross-section of the plug

= $(\rho g h) \times (\pi r^2)$

Here $\rho = 10^3 \text{ kg m}^{-3}$, $g = 10 \text{ ms}^{-2}$, $h = 15$ m, $r = 2.5$ cm $= 0.025$ m

$\Rightarrow f_s = (10^3 \times 10 \times 15) \times 3.14 \times (0.025)^2$

$= 0.0294 \times 10^4$

$= 294$ N

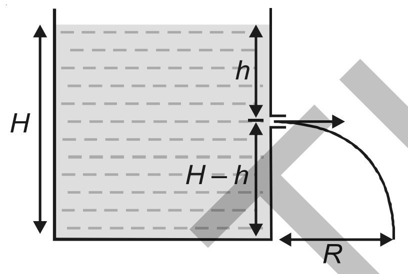
22. An open tank contains water upto height H. Its side wall contains an orifice at depth h below the water surface. What is the maximum range of the water flowing out?

Sol. Velocity of efflux of water $v_{eff} = \sqrt{2gh}$, this velocity is in horizontal direction.

$\therefore$ Vertical velocity of water when it leaves the orifice is zero.

The time taken by water to fall through height $H - h$ is given by $t = \sqrt{\frac{2(H-h)}{g}}$

The horizontal distance covered by water during this time



$R = v_{eff} \times t$

$$= \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{H(H-h)}$$

This is maximum for $h = \frac{H}{2}$

$\Rightarrow \quad R = \frac{H}{2}$

23. What happens when a capillary tube of insufficient length is dipped in a liquid?

Sol. When a capillary tube of insufficient length is dipped in a liquid, the liquid rises to the top. The radius of curvature of the concave meniscus increases till the pressure on its concave side becomes equal to that on its other side. But the liquid does not overflow.

24. What is Reynold's number? Give its significance.

Sol. Reynold's number is a dimensionless parameter which decides the nature of flow of a liquid through a pipe.

It is given by $R_e = \frac{\rho v D}{\eta}$.

If R_e lies between 0 to 1000, the flow is laminar, for $1000 \leq R_e \leq 2000$, the flow is unsteady, for $R_e > 2000$, the flow is turbulent.

25. Write the limitations of Bernoulli's principle.

Sol. → Bernoulli's equation ideally applies to non-viscous fluids having streamlined flow.

→ It is based on the assumption that there is no loss of energy due to friction.

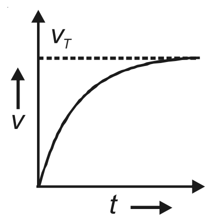
→ It is applicable to incompressible fluids only.

26. What is terminal velocity? Draw a curve that correctly shows the variation of velocity v with time t for a small spherical ball falling vertically in a long column of viscous liquid.

Sol. The maximum constant velocity acquired by a body while falling through a viscous medium is called its terminal velocity. For a ball of radius 'a' and density ρ falling through a fluid of viscosity η and density σ, the value of terminal velocity is given as

$$v_T = \frac{2a^2}{9\eta}(\rho - \sigma)g$$

Before attaining terminal velocity, the ball gets faster with time as shown in the following graph:



27. Two rain drops of the same radius fall through air with a steady velocity 5 cm/s. If the two drops coalesce, what is the terminal velocity of the new drop?

Sol. Terminal velocity $v_T = \frac{2a^2}{9\eta}(\rho - \sigma)g = 5 \times 10^{-2}$ m s^{-1} (initially)

When the two drops coalesce, the radius of the new drop becomes $R = \left(2^{\frac{1}{3}}\right)a$.

Hence new terminal velocity

$$v_T' = \frac{2R^2}{9\eta}(\rho - \sigma)g$$

$$= \frac{2\left(2^{\frac{2}{3}}\right)a^2}{9\eta}(\rho - \sigma)g$$

$$= \left(2^{\frac{2}{3}}\right)v_T$$

$$= \left(2^{\frac{2}{3}}\right) \times 5 \text{ cm s}^{-1}$$

$= 6.3$ cm s^{-1}

28. Give two examples of turbulent flow.

Sol. (i) When a fast-flowing stream encounters rocks, foamy whirlpool like regions are formed, which show turbulent flow.

(ii) The smoke rising up from a burning stack of wood is also an example of turbulence.

29. How do detergents enhance the cleansing action of water?

Sol. Molecules of detergents are hairpin shaped. Their one end is attracted to water and the other to the grease or oil. Thus, water grease interfaces are formed which decreases the surface tension of water. The greasy dirt is held suspended which otherwise is not made wet by water.

30. What is the excess pressure inside a liquid drop as compared to that in an air bubble of same radius?

Sol. A liquid drop has one free surface only. Hence, the excess pressure inside it is given by = $\frac{2S}{R}$

An air bubble has two free surfaces. Hence, for an air bubble of same radius, excess pressure inside it = $\frac{4S}{R}$. Thus, excess pressure inside a liquid drop is half of that in an air bubble of same radius.

31. What is the mathematical expression for capillary rise?

Sol. Capillary rise, $h = \frac{2S\cos\theta}{r\rho g}$

S = surface tension

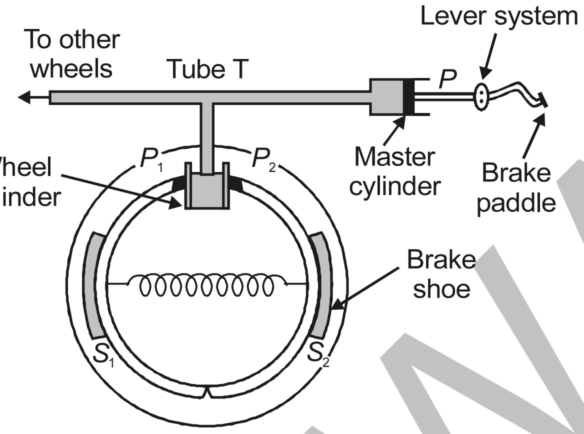
$\cos\theta$ = Angle of contact

Long Answer Type Questions :

32. Give the principle and explain the working of hydraulic brakes with a suitable diagram.

Sol. Hydraulic Brakes

Hydraulic brakes used in automobiles is based on Pascal's law. The wheels of the vehicle are connected to the brake pedal through an incompressible fluid called brake oil.



Hydraulic Brake

The piston directly connected to the brake paddle is called the master piston. When a little force is applied on the paddle with foot, the master piston moves inside. The pressure thus caused is transmitted undiminished to the other pistons which are directly connected to the wheels, (see figure). These pistons have smaller area of cross section. Applying Pascal's law, the braking forces produced at wheels is quite large as compared to the little force applied at the brake paddle.

One more advantage of using hydraulic brakes is that, an equal effect is produced at all the wheels of the vehicle. It is because pressure is transferred equally to all the pistons connected to the wheels. Thus, braking effort is equal on all wheels.

33. (i) Draw streamlines around a spinning ball.

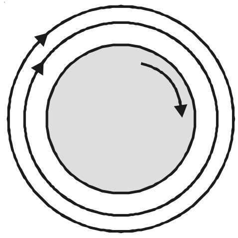
(ii) Explain Magnus effect.

(iii) Derive equation of continuity.

Sol. (i) and (ii)

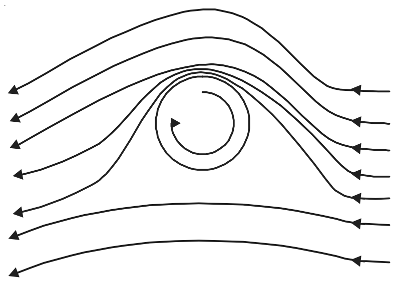
Magnus Effect

When a ball spins as it moves through air, it drags along a layer of air with it self. The streamlines around it are in the form of concentric circles as shown below.



When the ball moves forward, the air ahead of the ball rushes backward to fill the space left vacant by the ball. Thus streamlines of air around this ball are as shown below.

The velocity of air above the ball (in the given case) is opposite to the direction of spin. It is in the direction of spin below it.



Streamlines for a fluid around a sphere spinning clockwise

Thus, the relative velocity of air with respect to ball is greater at the upper part than that in the region below the ball. The streamlines thus get crowded above and rarified below.

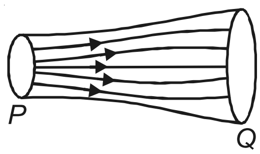
This causes the pressure below the ball to be greater and that above it to be smaller. This pressure difference causes an upward force on the ball. Hence, the ball is deviated from its path. This effect arising due to spinning of the ball is called "Magnus Effect".

(iii) **Equation of continuity**

Consider a fluid in steady flow. The map of its flow for a particular section can be shown by the bundle of streamlines as shown below. Here area of cross-section is greater at Q than that at P. Hence, streamlines are closely spaced at P than that at Q. Let the area of cross-section and fluid velocity at P be A_P and v_P respectively. Let the corresponding quantities at Q be A_Q and v_Q.

Therefore, the volume of fluid moving in at P, in a small time interval Δt

$$= A_P v_P \Delta t$$



Similarly, the volume flowing out at Q, during the same interval Δt,

$$= A_Q v_Q \Delta t$$

By conservation of mass, for Δt

mass flowing in at P = mass flowing out at Q.

$\Rightarrow (A_P v_P \Delta t)\rho_P = (A_Q v_Q \Delta t)\rho_Q$ [ρ_P = fluid density at P, ρ_Q = fluid density at Q]

For flow of incompressible fluids

$\rho_P = \rho_Q$

$\Rightarrow \boxed{A_p v_P = A_Q v_Q}$

This expression is known as **equation of continuity**. It is a statement of **conservation of mass** in flow for incompressible fluids.

The equation of continuity can be stated as : For the streamline flow of an incompressible fluid through a pipe of varying cross-section, Av remains constant through out the flow.

$\therefore Av$ = constant

34. State and prove Bernoulli's principle.

Sol. Bernoulli's Principle

This principle is based on the law of conservation of energy for fluids in motion. It may be stated as follows:

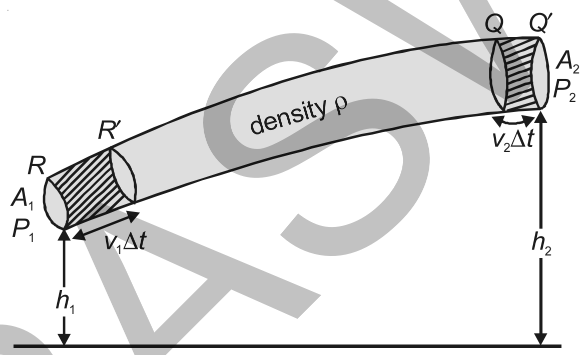
As we move along a streamline, the sum of the pressure (P), the kinetic energy per unit volume $\left(\frac{\rho v^2}{2}\right)$ and the potential energy per unit volume (ρgh) remains a constant.

Proof : According to the equation of continuity *i.e.*, Av = constant, fluid velocity changes with the change in cross-section. It means there must be some force acting on the fluid that causes this change in velocity. This resultant force arises due to the difference in pressure exerted by the fluid at different regions.

Consider the figure shown. It shows a portion of an incompressible fluid flowing steadily. Let the fluid initially be between the ends R and Q. Its velocity at end R is v_1 and that at Q is v_2. In an infinitesimal time interval Δt, fluid moves a distance $v_1\Delta t$ from R to R'. In the same time the fluid at the other end moves a distance $v_2\Delta t$ from Q to Q'.

Thus, the mass flowed in, in time Δt is

$\Delta m = (A_1 v_1 \Delta t)\rho = (A_2 v_2 \Delta t)\rho$



The pressure exerted by the oncoming fluid on this mass Δm at end R is P_1, and that at Q is P_2.

Thus, the force acting on the fluid inside the pipe at point R is $F_1 = P_1 A_1$

This force moves the fluid by a distance $v_1\Delta t$.

Hence, the work done by the force is

$W_1 = (F_1)(v_1\Delta t)$

$W_1 = P_1 A_1 v_1 \Delta t$

During the same interval Δt, the fluid inside the pipe pushes the fluid at point Q towards right by a distance $v_2\Delta t$.

Hence, the work done by the fluid inside the pipe

$W_2 = -P_2 A_2 v_2 \Delta t$

Negative sign appears because the work is done by the system.

So, the total work done on the fluid is

$W_1 + W_2 = P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t$

$= (P_1 - P_2)\Delta V$ ...(i) [By equation of continuity $A_1 v_1 \Delta t = A_2 v_2 \Delta t = \Delta V$]

This work done is related to the change in kinetic energy and change in its potential energy of the fluid by work energy theorem.

If the density of the fluid is ρ and $\Delta m = \rho A_1 v_1 \Delta t = \rho \Delta V$ is the mass passing through the pipe in time Δt, then change in its gravitational potential energy

$= \Delta mg(h_2 - h_1)$

$= (\rho\Delta V)g(h_2 - h_1)$ ...(ii)

Change in kinetic energy $= \frac{1}{2}\Delta m\left(v_2^2 - v_1^2\right)$

$= \frac{1}{2}(\rho\Delta V)\left(v_2^2 - v_1^2\right)$ ...(iii)

Using equations (i), (ii) and (iii); and applying work energy theorem,

$$(P_1 - P_2)\Delta V = g(h_2 - h_1)(\rho\Delta V) + \frac{1}{2}(\rho\Delta V)(v_2^2 - v_1^2)$$

$$\Rightarrow P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

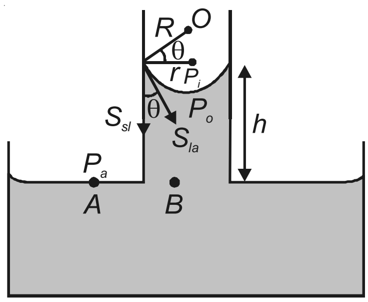
$$\Rightarrow \boxed{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}$$

In general form $\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$

This result is known as Bernoulli's principle.

35. Derive an expression for the rise of liquid in a capillary tube of uniform diameter and sufficient length.

Sol. Let a capillary tube of radius r be dipped in a liquid of surface tension S and density ρ. If the angle of contact for this liquid and the capillary tube is acute, the liquid forms a concave meniscus as shown. The curved part can be considered to be a part of an air bubble inside the liquid. Thus, pressure at a point just above the liquid surface is greater than that just below it. It is given by $P_i - P_o = \frac{2S}{R}$



Here R is the radius of the concave curved part.

As it is clear from the figure

$$R = \frac{r}{\cos\theta}$$

$$\therefore \quad P_i - P_o = \frac{2S\cos\theta}{r} \qquad \text{...(i)}$$

Since the region above the concave surface is open to the atmosphere, P_i = Atmospheric pressure P_a.

Also pressure at point A = atmospheric pressure P_a.

Since points A and B are at the same level

we have, $P_A = P_B = P_i$ ...(ii)

Using gauge pressure, pressure at point B is

$P_B = P_o + \rho gh$ ...(iii)

From equations (ii) and (iii), we have

$P_i = P_o + \rho gh$

or $P_i - P_o = \rho gh$

Substituting in equation (i) above

$$\rho gh = \frac{2S\cos\theta}{r}$$

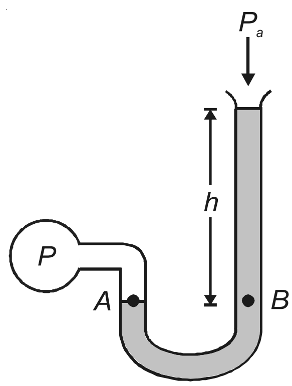
Thus, capillary rise $h = \dfrac{2S\cos\theta}{r\rho g}$.

If the liquid meniscus is convex, as in case of mercury, $\cos\theta$ is negative. Thus, h has a negative value. This tells that liquid will descend in this case.

36. What is an open tube manometer? Explain how does it work.

Sol. Open-Tube Manometer

An open-tube manometer is used for measuring pressure differences, or to measure the pressure of a gas enclosed in a vessel. It consists of a U-shaped tube open at both ends. A liquid of density ρ is filled in the tube. One end of the tube is left open to the atmosphere. The other end is connected with the system whose pressure is to be measured.



Due to the pressure exerted by the sample, and that by the atmosphere, the liquid in the U-tube rests at different levels in the two arms. Consider the two points A and B, as shown in the figure, lying at the same horizontal level. Then according to Pascal's law, pressures P_A and P_B at the respective points should be same. If h is the height of the liquid column above the point B, then

$P_A = P_B = P_a + \rho gh$

Here $P_A = P$ (pressure of the sample)

$\Rightarrow \boxed{P - P_a = \rho gh}$

What we normally measure is this gauge pressure *i.e.*, the difference between the sample pressure and the atmospheric pressure.

To measure small pressure differences, we use low density liquids like oil in the manometer. Whereas to measure large pressure differences, we use high density liquids like mercury.

37. Derive an expression for the velocity of efflux through an orifice in an enclosed container. Hence, derive Torricelli's law.

Sol. Speed of Efflux

The word 'efflux' means the outflow of the fluid. The expression for the velocity of efflux for a fluid, from a small hole of its container, can be found as follows.

The figure below shows a closed vessel filled with a liquid upto height *l*. Let the vessel contain a small hole (orifice) in its side at a level *h* below the top surface of liquid. Taking the liquid to be incompressible and its flow through the hole as streamline, we can apply the equation of continuity at points 1 and 2.

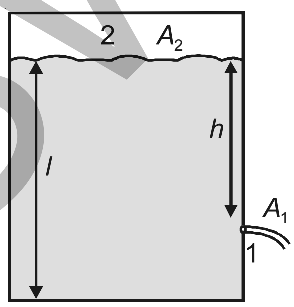
$A_2 v_2 = A_1 v_1$

$$\Rightarrow \boxed{v_2 = \frac{A_1 v_1}{A_2}} \qquad \text{...(i)}$$

Applying Bernoulli's equation at the two points, we have

$$P_2 + \frac{1}{2}\rho v_2^2 + \rho g l = P_1 + \frac{1}{2}\rho v_1^2 + \rho g(l - h)$$ [Point 1 is above the ground by a level $l - h$]

If the cross-sectional area of the vessel A_2 is much larger than that of hole *i.e.*, A_1



$v_2 \approx 0$

$$\Rightarrow \quad P_2 + \rho g l = P_1 + \frac{1}{2}\rho v_1^2 + \rho g(l - h)$$

$$\Rightarrow \quad \frac{1}{2}\rho v_1^2 = (P_2 - P_1) + \rho g h$$

$$v_1 = \sqrt{\frac{2}{\rho}\left[(P_2 - P_1) + \rho g h\right]}$$

Since the hole is open to the atmosphere, the pressure P_1 is same as the atmospheric pressure P_a.

$$\therefore \quad \boxed{v_1 = \sqrt{\frac{2}{\rho}(P_2 - P_a) + 2gh}}$$

This expression gives the velocity of efflux.

Torricelli's Law : In case the vessel containing the fluid is open *i.e.*, not covered, the pressure P_2 at the top of the liquid surface is same as the atmospheric pressure P_a.

Thus, the velocity of efflux becomes

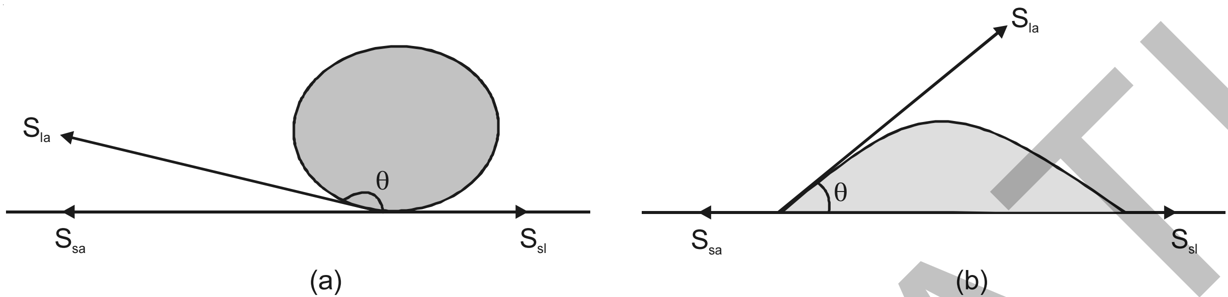
$$\boxed{v_{\text{eff}} = \sqrt{2gh}} \qquad \text{...(ii)}$$

This is the same as the velocity acquired by a body after falling freely through a height *h*.

The expression (ii) is known as Torricelli's law.

38. What are the factors on which the angle of contact depends? What does the value of angle of contact signify?

Sol. Angle of contact is defined as the angle that the tangent to a liquid surface at the point of contact makes with the solid surface inside the liquid. The angle of contact depends on the nature of the solid and the liquid in contact. At the line of contact, the surface forces between the three media must be in equilibrium.



Different shapes of water drops with interfacial tensions (a) on a lotus leaf (b) on a clean plastic plate.

(i) In the first case, the angle of contact θ is obtuse

Applying the condition of equilibrium for the three surface tensions at the point of contact, we get

$S_{sl} = S_{sa} + S_{la}\cos(\pi - \theta)$

$S_{sl} = S_{sa} - S_{la}\cos\theta$

$\Rightarrow S_{sl} > S_{sa}$

In this case liquid molecules are attracted strongly to themselves and weakly to those of solid. Hence, the liquid does not wet the solid surface.

For example, water on a greasy surface, and mercury on any surface. It is due to this that mercury splits into fine droplets when spread on our palm.

The surface of mercury when poured in a glass tube, bulges upward. We say it has a convex meniscus.

(ii) In the second case, the angle of contact is acute. Again applying the condition of equilibrium at the point of contact, we get $S_{sa} > S_{sl}$.

In such cases, liquid molecules are strongly attracted to those of solids and the liquid wets the solid.

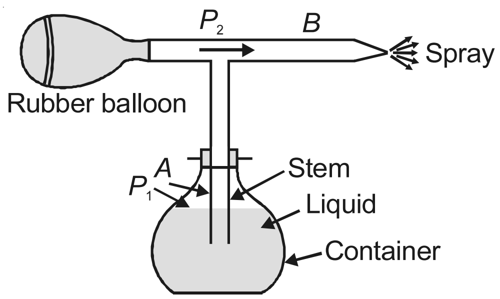
39. Explain three applications of Bernoulli's principle.

Sol. (i) **Carburettor of automobiles**

There is a venturi channel provided in the carburettor, Air flows through it at large speed. It lowers the pressure at narrow neck. Petrol rises up in the chamber due to this and provides a correct mixture of air and fuel.

(ii) **Atomiser or Sprayer**

It is another application of Bernoulli's principle. When the balloon is pressed, the air rushes out of the horizontal tube *B* decreasing the pressure in it. It causes the liquid in the bottle to rise up in the vertical tube. When it collides with the high speed air in tube *B*, it breaks up into fine spray.



(iii) **Streamlined shape given to bodies.**

Aircrafts and submarines are given special shapes called streamlined shapes so as to increase the dynamic lift. This is one of the applications of Bernoulli's principle.

40. (i) A wooden block floats in water with one third of its volume submerged. In oil, the block floats with two third of its volume submerged. Find the density of (a) wood, (b) oil, if the density of water is 10^3 kg m^{-3}.

(ii) A golden ornament weighs 60 g in air and 55 g in water. Assuming that some copper is mixed with gold to prepare the ornament, find the amount of copper in it. Given specific gravity of gold = 20, specific gravity of copper = 10.

Sol. (i) Let the density of block is ρ_b and the density of oil be ρ_o

Let the volume of block = V

In water

Block floats with one-third of its volume submerged, thus

weight of block = weight of water displaced

$$\Rightarrow \quad V\rho_b g = \frac{V}{3}\rho_{water} g$$

$$\Rightarrow \quad \rho_b = \frac{\rho_{water}}{3}$$

$$= \frac{10^3}{3}$$

$$= 333.3 \text{ kg m}^{-3}$$

In oil

Blocks floats with two third of its volume submerged.

$\Rightarrow$ wt. of block = wt. of oil displaced

$$\Rightarrow \quad V\rho_b g = \frac{2}{3}V\rho_o g$$

$$\Rightarrow \quad \rho_b = \frac{2}{3}\rho_o$$

$$\Rightarrow \quad \rho_{oil} = \frac{3}{2}\rho_b$$

$$= \frac{3}{2}\times 333.3$$

$$\approx 500 \text{ kg m}^{-3}$$

(ii) Let the amount of copper in the ornament = m

$\therefore$ The amount of gold in the ornament = $(60 - m)$

Density of gold, ρ_g = specific gravity of gold × 1 g cm^{-3}

= 20 g cm^{-3}

Similarly, Density of copper, $\rho_c = 10$ g cm^{-3}

Thus,

Volume of copper $V_1 = \frac{m}{10}$

Volume of gold $V_2 = \frac{60 - m}{20}$

When immersed in water

Decrease in weight = upthrust

$(60 - 55)g = (V_1 + V_2)\rho_w g$

$5 = \frac{m}{10} + \frac{60 - m}{20}$ [$\because \rho_w = 1$]

$5 = \frac{2m + 60 - m}{20}$

$100 = m + 60$

$m = 40$

$m = 40$ g

Thus, amount of copper in the ornament is 40 g.

41. How much energy is released when 8 identical water drops each of radius 0.5 cm combine to form one big drop? If the energy is converted to kinetic energy, find the velocity of the big drop.

[Surface tension of water = 0.07 N m^{-1}]

Sol. Given that radius of one small drop, $r = 0.5$ cm

$= 0.005$ m

8 such drops combine to form one big drop of radius R.

Then,

$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

$\Rightarrow R = 2r$

$= 2 \times 0.005$ m

$R = 0.01$ m

Now the energy released E, on combination of drops is given by

$E = S \times$ (decrease in area)

Where S is the surface tension of water, S = 0.07 N m^{-1}

$\Rightarrow E = 0.07[(4\pi r^2)8 - 4\pi R^2]$

$= 0.07 \times 4\pi[8r^2 - R^2]$

$= 0.07 \times 4 \times \frac{22}{7}\left[8(0.005)^2 - (0.01)^2\right]$

$= 0.88 \times (0.02 - 0.01) \times 10^{-2}$

$= 8.8 \times 10^{-5}$ J

This energy gets converted to kinetic energy,

$$\Rightarrow \; 8.8\times10^{-5} = \frac{1}{2}mv^2$$

Where m = mass of the drop, v = velocity of the bigger drop

$$\Rightarrow \quad 8.8\times10^{-5} = \frac{1}{2}\times\rho V\times v^2$$

$$\Rightarrow \quad 8.8\times10^{-5} = \frac{1}{2}(10)^3\left(\frac{4}{3}\pi\times10^{-6}\right)\times v^2$$

$$\Rightarrow \quad 4.2\times10^{-2} = v^2$$

$$\Rightarrow \quad v = 0.205 \text{ m/s}$$

42. A plane flies horizontally at constant speed. Each of its two wings has an area of 30 m^2. If the speed of the air is 150 km/h below the wing and 245 km/h above it, what is the plane's mass?

[Given air density = 1 kg m^{-3}; g = 9.8 ms^{-2}]

Sol. Given v_{lower} = 150 km/h = $150\times\frac{5}{18} = \frac{125}{3}\text{m s}^{-1}$

v_{upper} = 246 km/h = $246\times\frac{5}{18} = \frac{205}{3}\text{m s}^{-1}$

Total area of the two wings = 2 × 30 m^2 = 60 m^2; ρ = 1 kg m^{-3}

Applying Bernoulli's equation,

$$P_{lower} + \frac{1}{2}\rho v_{lower}^2 = P_{upper} + \frac{1}{2}\rho v_{upper}^2$$

$$P_{lower} - P_{upper} = \frac{1}{2}\rho\left(v_{upper}^2 - v_{lower}^2\right)$$

$$= \frac{1}{2}\times\frac{1}{9}\left(205^2 - 125^2\right)$$

$$= \frac{1}{18}(42025 - 15625)$$

$$= 1466.6 \text{ N m}^{-2}$$

Upward force on the plane = $(P_1 - P_2)\times A$

$= 1466.6 \times 60$

$= 88 \times 10^3$ N

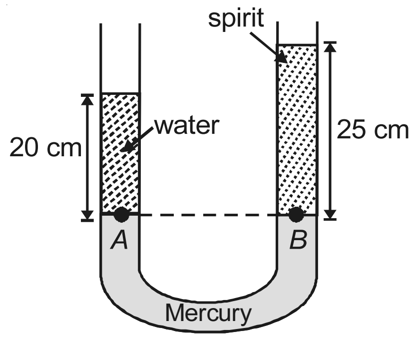
This force balances the weight of the plane.

Therefore $mg = 88\times10^3$

$$m = \frac{88\times10^3}{9.8}$$

Hence, the mass of the plane = 8979.5 kg

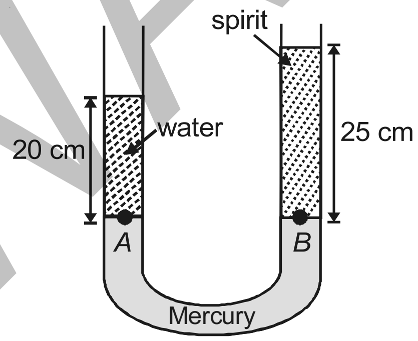
43. (i) A U-tube contains water and methylated spirit separated by mercury as shown below. What is the specific gravity of spirit?



(ii) In the above case if 10 cm of water and spirit each are further poured in the respective arms, find the difference of mercury levels in the two arms.

Sol. (i) The level of mercury is same in both the arms.

That means pressure at point A is the same as that at point B



$\Rightarrow \quad h_w \rho_w g = h_s \rho_s g$

$\Rightarrow \quad h_w \rho_w = h_s \rho_s$

$$\Rightarrow \quad \frac{\rho_s}{\rho_w} = \frac{h_w}{h_s}$$

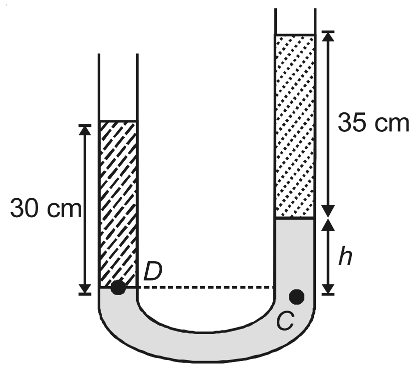
$$\frac{\rho_s}{\rho_w} = \frac{20\text{ cm}}{25\text{ cm}}$$

$\Rightarrow$ Specific gravity of spirit = 0.8.

(ii) When 10 cm of water and spirit each is poured in the respective arms, mercury level on the side of water goes down as water is denser than spirit.

Let the difference in the levels of mercury be h, now

equating the pressure at points C and D, we get



$h_w \rho_w g = h_s \rho_s g + h_m \rho_m g$

$\Rightarrow \quad h_w \rho_w = h_s \rho_s + h_m \rho_m$

$\Rightarrow \quad 30 \times 1 = 35 \times 0.8 + h \times 13.6$

$\Rightarrow \quad h = 0.147$ cm

$\boxed{h = 1.47\text{ mm}}$

44. (i) Distinguish between streamline flow and turbulent flow.

(ii) Show that Bernoulli's equation is same as the equation due to Pascal's law in presence of gravity, if a liquid or a gas is at rest.

Sol. (i) **Streamline Flow**

1. In a streamline flow each following (oncoming) particle follows exactly the same path as that of its predecessor.

2. Flow is steady and different layers of liquid move parallel to each other.

3. Reynolds number is usually less than <1000 for a streamline flow.
4. Fluid velocity remains constant at any point of a streamline, but it may be different at different points of the same streamline.
5. It occurs at low speeds.
6. A streamline motion can be represented with help of a bundle of streamlines.

Turbulent Flow:

1. The haphazard and zig-zag flow of fluid particles is called turbulent flow.
2. It is accompanied by random irregular local circular currents called eddies.
3. It occurs at high flow speeds.
4. Mostly Reynolds number > 2000 for the flow to be turbulent.
5. It leads to dissipation of kinetic energy as heat.
6. Turbulent motion increases the rate of transfer of mass, momentum and energy.

(ii) According to Bernoulli's principle, $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$. If the fluid is at rest,

$\Rightarrow v_1 = v_2 = 0$

Then, $P_1 + \rho g h_1 = P_2 + \rho g h_2 \Rightarrow \boxed{P_1 - P_2 = \rho g(h_2 - h_1)}$

Which is a mathematical form of Pascal's law, *i.e.*, at $h_1 = h_2$, $P_1 - P_2 = 0$

45. Explain why

(i) Mercury does not wet glass

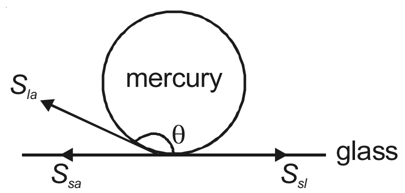
(ii) The hair of a paint brush cling together when taken out of water

(iii) Heavier drops flatten and are not spherical in shape.

Sol. (i) The angle of contact θ for mercury-glass interface is obtuse. The molecules of mercury are strongly attracted to themselves than that to glass molecules.

Hence mercury does not wet glass.

Applying the condition of equilibrium at the point of contact, we have



$S_{sa} + S_{la}\cos(\pi - \theta) = S_{sl}$

$\Rightarrow S_{sl} = S_{sa} - S_{la}\cos\theta$

Since θ is obtuse, $S_{sl} > S_{sa}$

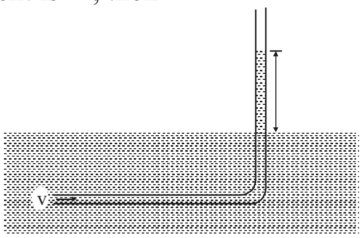
(ii) When brush is taken out of water, thin water film is formed at the tips of the hair. It contracts due to surface tension and so the hair cling together.

(iii) In absence of any external force, a liquid surface acquires a spherical shape due to surface tension. It is because for a given volume, a sphere has the minimum surface area.

Therefore, small liquid drops are spherical in shape, large drops get flattened due to the action of gravity.

Multiple Choice Questions

1. An L-shaped glass tube is just immersed in flowing water such that its opening is pointing against flowing water. If the speed of water current is v, then



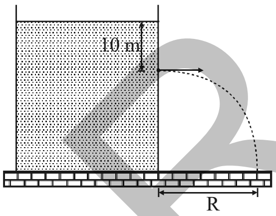
(a) The water in the tube rises to height $\frac{v^2}{2g}$

(b) The water in the tube rises to height $\frac{g}{2v^2}$

(c) The water in the tube does not rise at all

(d) None of these

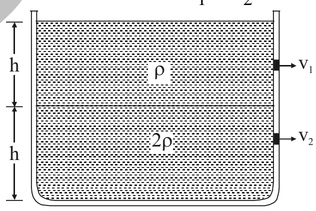
2. A large tank is filled with water 9density $=10^3 kg/m^3$. A small hole is made at a depth 10 m below water surface. The range of water issuing out of the hole is R on ground. What extra pressure must be applied on the water surface so that the range becomes 2R (take 1 atm $=10^5\ Pa$ and $g=10\ m/s^2$)



(a) 9 atm (b) 4 atm

(c) 5 atm (d) 3atm

3. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in figure. Two small holes are punched at depth h/2 and 3h/3 from the surface of lighter liquid. If v_1 and v_2 are the velocities of a flux at these two holes, then v_1/v_2 is



(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{2}}$

4. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water the quantities of water flowing out per second from both the holes are the same. Then R is equal to

(a) $2\pi L$ (b) $\frac{L}{\sqrt{2\pi}}$

(c) L (d) $\frac{L}{2\pi}$

5. Water is flowing continuously from a tap having an internal diameter $8\times10^{-3} m$. The water velocity as it leaves the tap is $0.4\ ms^{-1}$. The diameter of the water stream at a distance $2\times10^{-1} m$ below the tap is close to :

(a) $7.5\times10^{-3}\ m$ (b) $9.6\times10^{-3} m$

(c) $3.6\times10^{-3}\ m$ (d) $5.0\times10^{-3}\ m$

6. Water is flowing through a horizontal tube having cross-sectional areas of its two ends being A and A' such that the ration A/A' is 5. If the pressure difference of water between the two ends is $3\times10^5\ Nm^{-2}$, the velocity of water with which it enters the tube will be (neglect gravity effects)

(a) $5\ ms^{-1}$ (b) $10\ ms^{-1}$

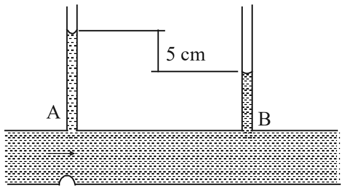
(c) $25\ ms^{-1}$ (d) $50\sqrt{10}\ ms^{-1}$

7. Air of density $1.2\ kgm^{-3}$ is blowing across the horizontal wings of an aeroplane in such a way that its speeds above and below the wings are $150\ ms^{-1}$ and $100\ ms^{-1}$, respectively. The pressure difference between the upper and lower sides of the wings, is :

(a) $60\ Nm^{-2}$ (b) $180\ Nm^{-2}$

(c) $7500\ Nm^{-2}$ (d) $12500\ Nm^{-2}$

8. In the diagram shown the difference in the two tubes of the manometer is 5 cm, the cross section of the tube at A and B is $6\,mm^2$ and $10\ mm^2$ respectively. the rate at which water flows through the tube is $\left(g=10\ ms^{-2}\right)$

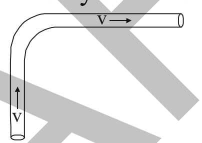


(a) 7.5 cc/s (b) 8.0 cc/s
(c) 10.0 cc/s (d) 12.5 cc/s

9. A barometer kept in an elevator accelerating upwards reads 76 cm of Hg. What will be the possible air pressure inside the elevator ?
(a) equal to 76 cm of Hg
(b) less than 76 cm of Hg
(c) greater than 76 cm of Hg
(d) zero

10. A bowl full of water has a bottom area $4cm^2$, and height 20 cm. The volume of water in it is one liter. the force exerted by the sides of bowl on water is :
(a) 10 N (b) 8 N
(c) 2 N (d) 4 N

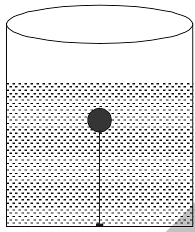
11. A fire hydran delivers water of density ρ at a volume rate Q. The water travels vertically upward through the hydrant and then does $90°$ turn to emerge horizontally at speed v. The pipe and nozzle have uniform cross-section throughout. The force exerted by the water on the corner of the hydrant is:



(a) ρvQ (b) zero
(c) $2\rho vQ$ (d) $\sqrt{2}\rho vQ$

12. A cubical block of wood 20.0 cm on a side and density of $500\ kg/m^3$ floating on water. From its equilibrium floating position, it is pushed further by 4.0 cm into the water. What is the force needed to keep the block in this new position ? $g=10\ m/s^2$
(a) 16 N (b) 32 N
(c) 40 N (d) 56 N

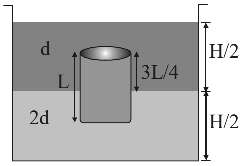
13. A solid sphere of density $\eta(>1)$ times lighter than water is suspended in a water tank by a string tied to its base as shown in fig. If the mass of the sphere is m, then the tension in the string is given by :



(a) $\left(\frac{\eta-1}{\eta}\right)$mg (b) ηmg

(c) $\frac{mg}{\eta-1}$ (d) $(\eta-1)mg$

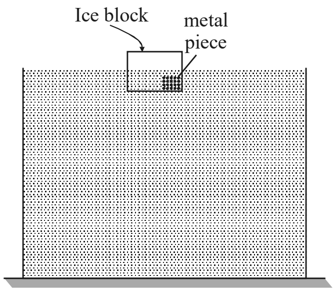
14. A homogeneous solid cylinder of length $L(L<H/2)$. Cross-sectional area A/5 is immersed such that it floats with its axis vertical at the liquid-liquid interface with length L/4 in the denser liquid as shown in the fig. The lower density liquid is open to atmosphere having pressure P_0. The density D of solid is given by :



(a) $\frac{5}{4}d$ (b) $\frac{4}{5}d$

(c) d (d) $\frac{d}{5}$

15. A small metal piece is inside an ice block which floating in a container full of water. The level of water in the container when ice melts :



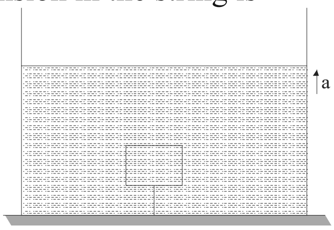
(a) same (b) increase
(c) decrease (d) nothing can be solid

16. Two communicating vessels contain mercury. The diameter of one vessel is n times larger than the diameter of the other. A column of water of height h is poured into the left vessel. The mercury level will rise in the right-hand vessel (s=relative density of mercury and ρ = density of water) by

(a) $\frac{n^2h}{(n+1)^2 s}$ (b) $\frac{h}{(n^2+1)s}$

(c) $\frac{h}{(n+1)^2 s}$ (d) $\frac{h}{n^2 s}$

17. A body having volume V and density ρ is attached to the bottom of a container as shown. Density of the liquid is $d(>\rho)$. Container has a constant upward acceleration a. Tension in the string is



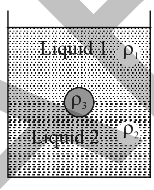
(a) $V\left[dg-\rho(g+a)\right]$

(b) $V\left[(g+a)(d-\rho)\right]$

(c) $V(d-\rho)g$

(d) none

18. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball made of a material of density ρ_3 is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3



(a) $\rho_1 > \rho_3 > \rho_2$ (b) $\rho_1 > \rho_2 > \rho_3$

(c) $\rho_1 < \rho_3 < \rho_2$ (d) $\rho_3 < \rho_1 < \rho_2$

19. A fluid comparises of

(a) liquid only

(b) gases only

(c) Neither liquid nor gases

(d) Both (a) and (b)

20. Which of the following is correct.

(a) Gauge pressure = absolute pressure + atmospheric pressure

(b) Absolute pressure = gauge pressure – atmospheric pressure

(c) Gauge pressure = absolute pressure – atmospheric pressure

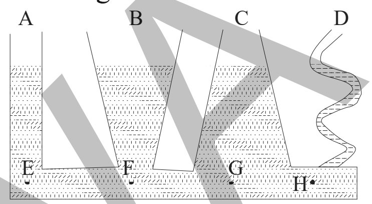
(d) Absolute pressure = atmospheric pressure – gauge pressure

21. At a depth of 550 m in an ocean, what is the absolute pressure? Given that the density of sea water is 1.03×10^3 kg m^3 and $g = 10$ ms^{-2}.

(a) 40 atm (b) 52 atm

(c) 32 atm (d) 62 atm

22. Four vessels A, B, C and D have different shapes and hold different amount of water. Which of the following is correct.



(a) $P_E > P_F > P_G > P_H$

(b) $P_E < P_F < P_G < P_H$

(c) $P_E = P_F = P_G = P_H$

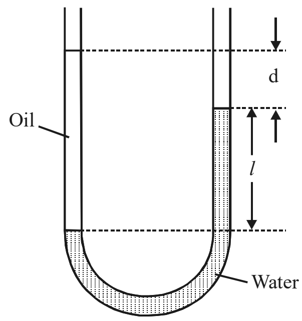
(d) $P_E = P_F > P_G = P_H$

23. Two liquids of densities 2ρ and ρ having their volumes in the ratio 3 : 2 are mixed together. Density of the mixture will be

(a) $\frac{2\rho}{3}$ (b) $\frac{\rho}{2}$

(c) $\frac{8\rho}{5}$ (d) $\frac{4\rho}{5}$

24. The U-tube in figure contains two different liquids in static equilibrium, water in the right arm and oil of unknown density ρ_x in the left. If $I = 135$ mm and $d = 15$ mm. Density of the oil is



(a) 1000 kg m^3 (b) 920 kg m^3

(c) 895 kg m^3 (d) 900 kg m^3

25. Increase in pressure at one point of the enclosed liquid in equilibrium at rest is transmitted equally to all other points. This is as per

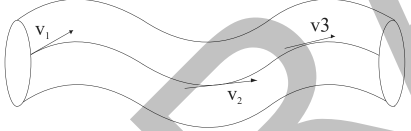
(a) impulse
(b) Pascal's law
(c) conservation of momentum
(d) None of the above

26. In a vehicle lifter the enclosed gas exerts a force F on a small piston having a diameter of 8 cm. This pressure is transmitted to a second piston of diameter 24 cm. If the mass of the vehicle to be lifted is 1400 kg then value of F is
(a) 1200 N (b) 1800 N
(c) 1600 N (d) 700 N

27. A boat floating in a water tank is carrying a number of stones. If the stones were unloaded into water, then the water level
(a) increases (b) remains same
(c) decreases (d) data is insufficient

28. An iron casting containing a number cavities weights 6000 N in air and 4000 N in water. What is the volume of the cavities in the casting? Density of iron is $7.87\ gcm^{-3}$ (Take, $g = 9.8\ ms^{-2}$ and density of water $= 10^3\ kgm^{-3}$)
(a) $0.16\ m^3$ (b) $0.2 m^3$
(c) $0.12\ m^3$ (d) $0.14\ m^3$

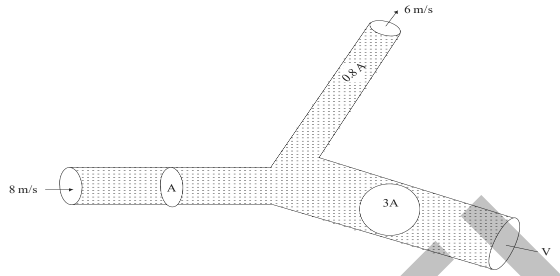
29. Consider streamline flow of a liquid flowing through a tube as shown in the figure which of the following is correct regarding velocities of liquid at different points?



(a) $V_1 =$ constant, $V_2 =$ constant, $V_3 =$ constant
(b) $V_1 \neq V_2 \neq V_3$
(c) $V_1 = V_2 = V_3$
(d) Both (a) and (b) are current

30. If R_e is the Reyonld's number, then which of the following is incorrect
(a) For R_e flow is laminar
(b) For $1000 < R_e < 2000$, flow is steady
(c) For $R_e > 2000$ flow is turbulent
(d) All are incorrect

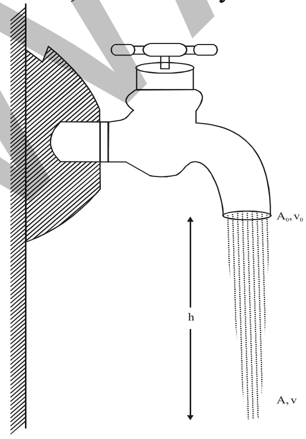
31. An incompressible liquid is flowing through a horizontal pipe as shown in figure. The value of speed V is



(a) $1.1\ ms^{-1}$ (b) $2.1\ ms^{-1}$
(c) $3.1\ ms^{-1}$ (d) $5.1\ ms^{-1}$

32. Bernoulli's theorem is a consequence of
(a) conservation of mass
(b) conservation of energy
(c) conservation of linear momentum
(d) conservation of angular momentum

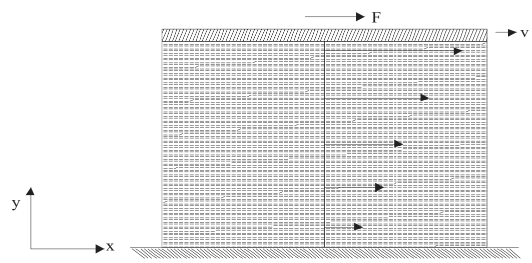
33. Water falls from a tap with $A_0 = 4\ m^2, A = 1\ m^2$ and $h = 2$ m, then velocity V is



(a) $2.5\ ms^{-1}$ (b) $6.5\ ms^{-1}$
(c) $4.5\ ms^{-1}$ (d) $1.5\ ms^{-1}$

34. The velocity of efflux of a liquid through an orifice in the bottom of a tank does not depend upon
(a) density of liquid
(b) height of the liquid column above orifice
(c) acceleration due to gravity
(d) None of the above

35. A tank is filled to a height H. The range of water coming out of a hole which is a depth $H/4$ from the surface of water level is



(a) $\frac{2H}{\sqrt{3}}$ (b) $\frac{\sqrt{3}H}{2}$
(c) $\sqrt{3}H$ (d) $\frac{3H}{4}$

36. Units of coefficient of viscosity are
(a) Nms^{-1} (b) NM^2S^{-1}
(c) $Nm^{-2}s$ (d) None of these

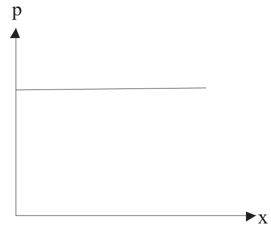
37. The rate of flow of liquid in a tube of radius r, length I, whose ends are maintained at a pressure difference ρ is $V = \frac{\pi N p r^4}{\eta l}$, where η is coefficient of the viscosity and N is
(a) 8 (b) $\frac{1}{8}$
(c) 16 (d) $\frac{1}{16}$

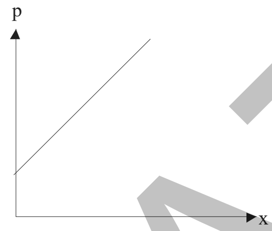
38. Two capillary tubes of the same length but different radii r_1 and r_2 are fitted in parallel to the bottom of a vessel. The pressure head is ρ. What should be the radius of a single tube that can replace the two tubes so that the rate of flow is same as before
(a) $r_1 + r_2$ (b) $\frac{r_1 r_2}{r_1 + r_2}$
(c) $\frac{r_1 + r_2}{2}$ (d) None of these

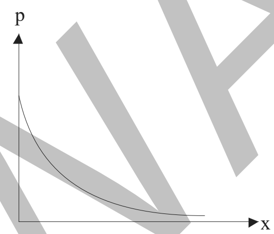
39. An air bubble rises from the bottom of a lake of large depth. The rising speed of air bubble will
(a) go on increasing till it reaches surface
(b) go on decreasing till it reaches surface
(c) increases in two beginning, then will become constant
(d) be constant all throughout

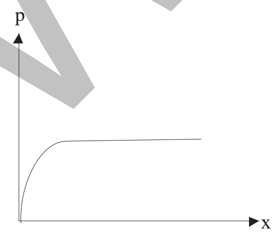
40. Two equal drops of water are falling through air with a steady velocity V. If the drops coalesce, the new velocity will
(a) $2V$ (b) $\sqrt{2}V$
(c) $2^{2/3}V$ (d) $\frac{V}{\sqrt{2}}$

41. From amongst the following curves, which one shows the variation of the velocity V with time t for a small sized spherical body falling vertically in a long column of a viscous liquid

(a)


(b)


(c)


(d)


42. A spherical ball of radius 3.0×10^{-4} m and density 10^4 kgm^{-3} falls freely uner gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, what is the value of h (Viscosity of water is 9.8×10^{-6} Nsm^{-2}).
(a) 1.65×10^3 m (b) 2.65×10^2 m
(c) 3.65×10^4 m (d) 1.45×10^2 m

43. Which of the following is not the unit of surface tension?
(a) newton/metre (b) joule/$(\text{metre})^2$
(c) $\text{kg}(\text{second})^2$ (d) watt/metre

44. If work W is done is blowing a bubble of radius R from soap solution, then the work done in blowing a bubble of radius $2R$ from the same solution is
(a) $W/2$ (b) $2W$
(c) $4W$ (d) $2\frac{1}{3}W$

45. If two soaps bubble of equal radii r coalesce, then the radius of curvature of interface between two bubble will be

(a) r (b) 0

(c) infinity (d) $\frac{r}{2}$

46. Water rises in a capillary tube to a height of 2.0 cm. in another capillary tube whose radius is one-third of it, how much the water will rise?

(a) 6.0 cm (b) 2.0 cm

(c) 4.0 cm (d) 8.0 cm

47. The water droplets in free fall are spherical due to

(a) gravity

(b) viscosity

(c) surface tension

(d) inter-molecular attraction

48. Hair of shaving brush cling together when it is removed from water due to

(a) force of attraction between hair

(b) surface tension

(c) viscosity of water

(d) characteristic property of hair

49. Small droplets of liquid are usually more spherical in shape than larger drops of the same liquid because

(a) force of surface tension is equal and opposite to the force of gravity

(b) force of surface tension predominates the force of gravity

(c) force of gravity predominates the force of surface tension

(d) force of gravity and force of surface tension act in the same direction and are equal

50. Water does not wet an oily glass because

(a) cohesive force of oil > adhesive force between oil and glass

(b) cohesive force of oil > cohesive force of water

(c) oil repels water

(d) cohesive force of water > adhesive force between water and oil molecules

Answer Key

1. A	2. D	3. D
4. B	5. C	6. A
7. C	8. A	9. C
10. C	11. D	12. A
13. D	14. A	15. C
16. B	17. B	18. C
19. D	20. C	21. B
22. C	23. C	24. D
25. B	26. C	27. C
28. C	29. D	30. B
31. A	32. B	33. B
34. A	35. B	36. C
37. D	38. D	39. C
40. C	41. D	42. A
43. D	44. C	45. C
46. A	47. C	48. B
49. B	50. D	